Major and Minor Ellipse Axes from a diameter and two Lengths

The diagram below presents geometry for the inside of a conical - elliptic bore. It's purpose is to derive major and minor axes corresponding to measurements of the form:

(d, x, y)

where	d	is the distance between the sides of the bore at either x or y
	x	is the length into the bore when the distance is measured along a plane perpendicular to a line through the centre of the tone holes
	y	is the length into the bore when the distance is measured along a plane parallel to a line through the centre of the tone holes

Because a single tuple in the above form is not sufficient to describe the angle of conicity of the bore, two such tuples are required i.e. (d_1, x_1, y_1) (d_2, x_2, y_2). In the following diagram the second measurement tuple is closer to the wider end of the bore than the first.

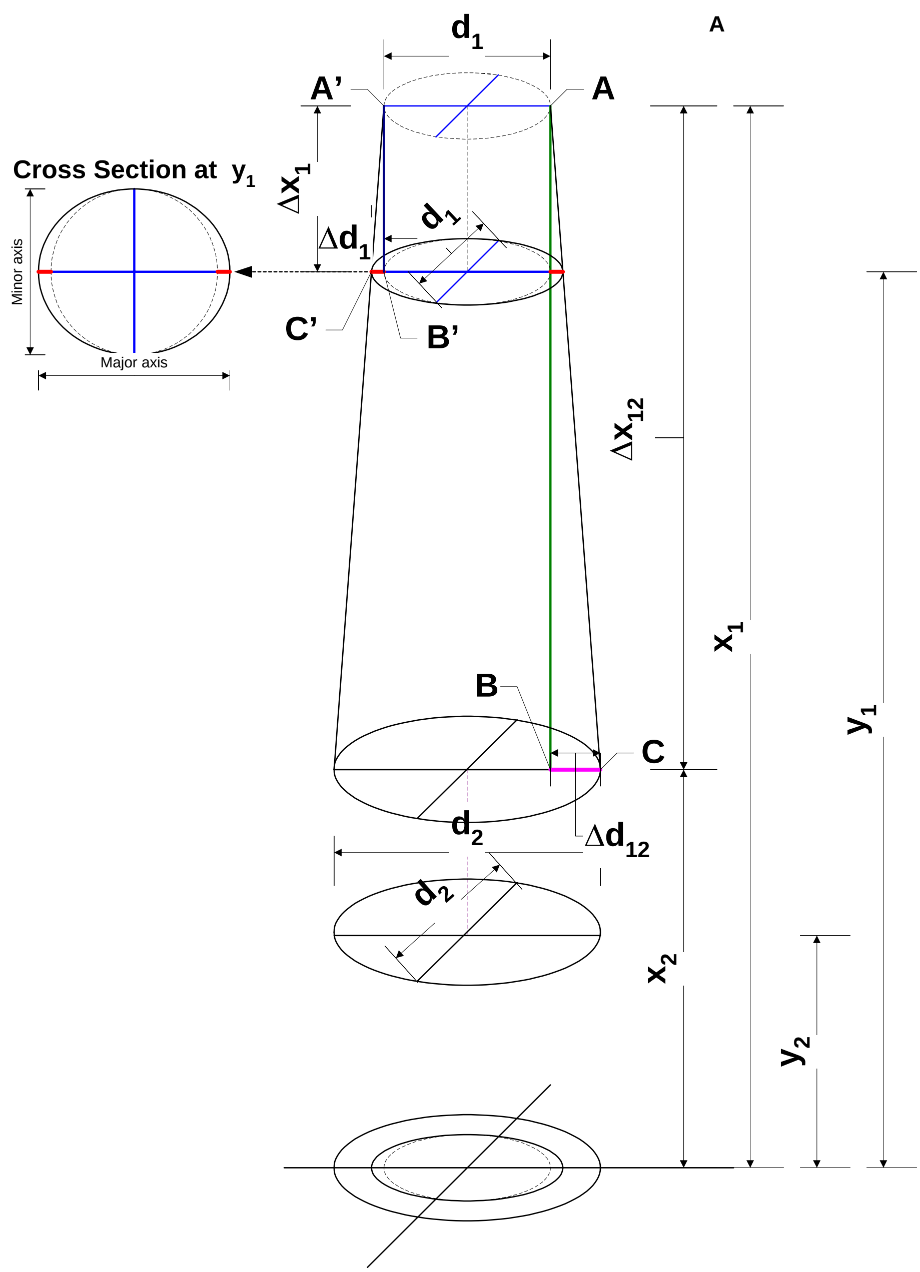

Figure 1: Bore Geometry

In the above, the following relationships are true:

$$x_1 > y_1$$

$$d_2 > d_1$$

$$y_1 > y_2$$

$$x_1 > x_2$$

The goal of the exercise is to find an expression for Δd_1. We make use of the fact that the triangles ABC and A'B'C' are similar i.e. all angles are the same. Based on this, the following is true:

$$\frac{\Delta d_1}{\Delta x_1} = \frac{\Delta d_{12}}{\Delta x_{12}} \quad \text{or} \quad \frac{\Delta d_1}{(x_1 - y_1)} = \frac{(d_2 - d_1)}{(x_1 - x_2)}$$

Formula 1:

Solving for Δd_1 (We know everything else):

$$\Delta d_1 = \frac{(d_2 - d_1) \cdot (x_1 - y_1)}{(x_1 - x_2)}$$

Formula 2:

The major and minor axes (M and m) at y_1 are:

$$M = d_1 + \frac{(d_2 - d_1) \cdot (x_1 - y_1)}{(x_1 - x_2)} \quad \text{or} \quad M = \frac{d_1 \cdot (x_1 - x_2) + (d_2 - d_1) \cdot (x_1 - y_1)}{(x_1 - x_2)}$$

$$m = d_1$$

Formula 3:

Similar expressions may derived for ellipses at x_1, x_2 and y_2.